7: Bayes Factors and Hypothesis Testing

Bayesian Mixed Effects Models with brms for Linguists

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Table of contents

1 The Question: Which Hypothesis is Better Supported?

1.1 Research Scenario

You've estimated your model parameters and found that an effect exists (Module 06). Now you want to answer a different question:

“How much evidence do the data provide for one hypothesis over another?”

Examples:

- H : No effect (= 0) vs. H : Some effect (0)
- H : Positive effect (> 0) vs. H : Negative effect (< 0)
- Model A: Simple main effects vs. Model B: Include interaction

This is where **Bayes Factors** come in.

1.2 Bayes Factor vs. ROPE: Different Questions

Approach	Question	Output	Focus
ROPE (Module 06)	“Is the effect meaningful?”	Accept/Reject/Undecided	Practical significance
Bayes Factor (Module 07)	“Which hypothesis is better?”	Evidence ratio (e.g., 10:1)	Hypothesis comparison

Key insight: These are complementary, not competing!

- Use ROPE when you care about **practical significance**
- Use Bayes Factors when comparing **competing hypotheses/theories**

1.3 Today

1. **`hypothesis()` function**: Compute Bayes Factors for parameter constraints
    - Uses Savage-Dickey density ratio method
    - Fast and built into brms
2. **`bayes_factor()` function**: Compare full models
    - Uses bridge sampling (more general but slower)
    - For complex model comparisons
3. **Interpretation guidelines**: What does BF = 3 vs. BF = 30 mean?

2 Where We Are in the Analysis Workflow

2.1 The Bayesian Workflow So Far

```
Module 01-02: Build model + set priors
            ↓
Module 03: Check model fit (posterior predictive)
            ↓
Module 04: Test prior sensitivity
            ↓
Module 05: Compare models (LOO-CV for prediction)
            ↓
Module 06: Practical significance (ROPE, emmeans, marginaleffects)
            ↓
Module 07 (TODAY): Hypothesis comparison (Bayes Factors, hypothesis())
```

2.2 LOO vs. Bayes Factors vs. ROPE: What's the Difference?

These three approaches address different aspects of model evaluation:

LOO (Module 05):

- Goal: Predictive accuracy
- Question: "Which model predicts new data better?"
- Method: Cross-validation (leave-one-out)
- Output: ELPD difference, standard errors
- Use when: You care about out-of-sample performance
- Example: Choose between polynomial degrees for smooth fit

Bayes Factors (Module 07):

- Goal: Relative evidence
- Question: "Which hypothesis is better supported by data?"
- Method: Ratio of marginal likelihoods
- Output: Evidence ratio (e.g., BF   = 10:1)
- Use when: You want to quantify evidence for competing theories
- Example: Does priming effect exist? (H vs. H)

ROPE (Module 06):

- Goal: Practical significance
- Question: "Is the effect too small to matter?"
- Method: Region of Practical Equivalence + HDI
- Output: Accept/Reject/Undecided
- Use when: You need to decide if an effect is meaningful
- Example: Is the treatment effect large enough to be clinically relevant?

Rule of thumb:

- Use **LOO** for model selection when **prediction** matters
- Use **BF** for hypothesis comparison when **explanation/theory** matters

- Use **ROPE** for decision-making when **practical significance** matters
- Use **all three together** for a complete analysis!

The Mathematical Relationship: Can These Approaches Be "Reverse Engineered"?

Recent research has explored the mathematical relationships between **Bayes Factors, HDI-ROPE, and frequentist equivalence tests (TOST)**.
Note: LOO is NOT part of this discussion because it addresses a fundamentally different question:

- **LOO**: Predictive accuracy (out-of-sample performance)
- **BF/ROPE/TOST**: Evidence for hypotheses (in-sample inference about parameters)

These are complementary, not competing. The "reverse engineering" debate concerns only methods testing the same hypothesis (equivalence/existence) using different frameworks.

2.2.1 Campbell & Gustafson (2022)

"The Bayes Factor, HDI-ROPE, and Frequentist Equivalence Tests Can All Be Reverse Engineered"
Campbell and Gustafson (2022) redid the simulation study of Linde et al. (2021) with a critical modification: **they calibrated all three procedures to have the same predetermined maximum Type I error rate**.
Main conclusions:

1. **Similar operating characteristics**: When calibrated to the same Type I error rate, Bayes Factors, HDI-ROPE, and frequentist equivalence tests (TOST) all have **almost identical Type II error rates**.
2. **Methods are interchangeable (mathematically)**: The three approaches can be "reverse-engineered" from one another – they're essentially different mathematical frameworks achieving the same goal.
3. **Philosophy matters more than performance**: "If one decides on which underlying principle to subscribe to in tackling a given problem, then the method follows naturally."
4. **Empirical comparison is futile**: "Trying to use empirical performance to argue for one approach over another seems like tilting at windmills."

2.2.2 Linde et al. (2023)

"Decisions about Equivalence: A Comparison of TOST, HDI-ROPE, and the Bayes Factor"
Linde et al. (2023) conducted an extensive simulation comparing operating characteristics across various scenarios.
Main conclusions:

1. **Bayes Factor shows advantages**: The Bayes Factor interval null approach performed best in their simulations for:
   - Distinguishing between equivalence and non-equivalence
   - Maintaining control over error rates
   - Flexibility across different scenarios
2. **HDI-ROPE is more conservative**: HDI-ROPE tends to be more conservative (lower Type I errors, but higher Type II errors) compared to Bayes Factors.
3. **TOST depends heavily on sample size**: Frequentist TOST can struggle with small samples and requires careful power analysis.
4. **Recommendation**: "Researchers rely more on the Bayes factor interval null approach for quantifying evidence for equivalence."

2.2.3 Practical Implications for Your Analysis

What this means:

1. **Don't agonize over method choice (for equivalence testing)**: If calibrated properly, BF and ROPE give similar conclusions. Choose based on your philosophical stance and audience.
2. **Report both when possible**: Since they're complementary perspectives on the same question, reporting both BF and ROPE provides fuller picture:
   - **BF**: "How much evidence?" (continuous quantification)
   - **ROPE**: "Is it negligible?" (decision rule)
3. **Focus on calibration**: The key is proper calibration of thresholds:

    - **ROPE width**: Should reflect your domain's practical significance
    - **BF threshold**: Should match your evidence requirements
    - **Prior specification**: Critical for both approaches
4. **Philosophy guides choice**:
    - **Bayesian mindset** → Use BF for evidence quantification
    - **Decision-focused** → Use ROPE for accept/reject framework
    - **Mixed audience** → Report both

Example of integrated reporting:

> "We compared models using both LOO (predictive accuracy) and Bayes Factors (evidence). The random slopes model showed better predictive performance (ΔELPD = 12.3, SE = 4.2) and strong evidence for the complexity effect (BF = 24.5). The 95% HDI [0.08, 0.15] fell entirely outside our ROPE of ±0.03, indicating the effect is predictively useful, evidentially supported, and practically meaningful."

This gives: - **Predictive quality** (LOO: ΔELPD = 12.3) - **Evidence strength** (BF = 24.5) - **Practical significance** (HDI outside ROPE) - **Effect size** (HDI: [0.08, 0.15])

3 The Savage-Dickey Density Ratio Method (brms's hypothesis())

3.1 What is the Savage-Dickey Method?

The **Savage-Dickey method** provides an elegant way to compute Bayes Factors when:

- Models are **nested** (one is a special case of the other)
- You're testing a point hypothesis (e.g., = 0)

The formula:

$$BF_{01} = \frac{p(\theta = \theta_0 | \text{Data}, H_1)}{p(\theta = \theta_0 | H_1)} = \frac{\text{posterior density at null}}{\text{prior density at null}}$$

Intuition:

- If data make null value **more plausible** → BF > 1 → evidence for H
- If data make null value **less plausible** → BF < 1 → evidence for H

3.2 Visual Understanding

Interpretation of this example:

- Prior density at = 0: 0.402
- Posterior density at = 0: 0.356
- The posterior density **decreased** at the null
- BF = 1.13 → Data are ~1.1 times more likely under H
- This is **positive evidence** for an effect

3.3 Why This Works (Technical)

For nested models where H : = is a special case of H : p( ):

$$BF_{01} = \frac{p(\text{Data}|H_0)}{p(\text{Data}|H_1)}$$

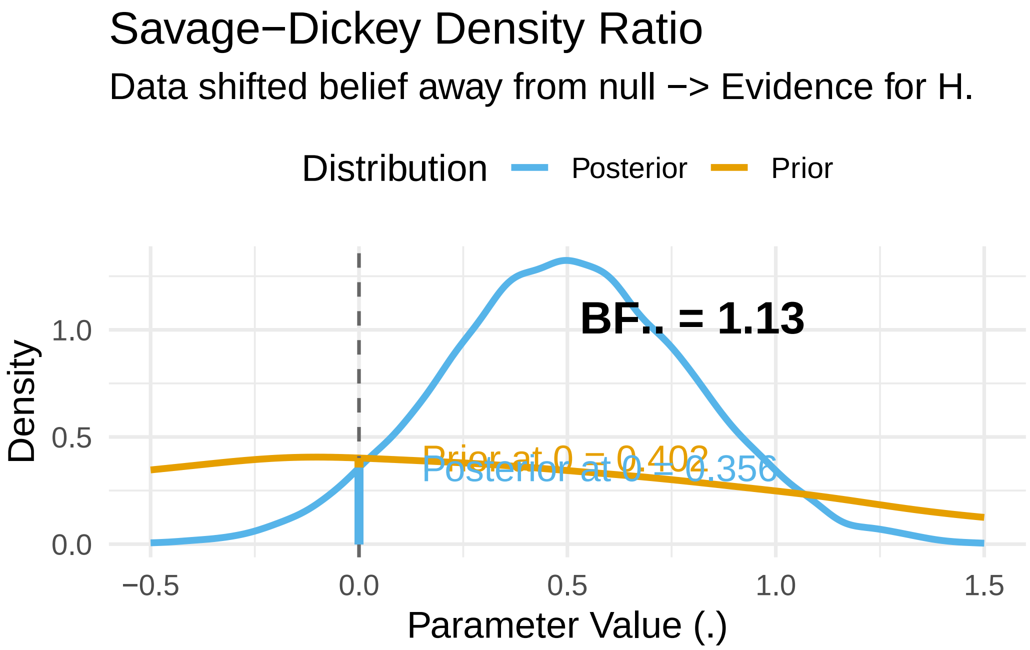


Figure 1: Savage-Dickey Density Ratio Illustration

The Savage-Dickey method shows that this equals:

$$BF_{01} = \frac{p(\theta = \theta_0 | \text{Data}, H_1)}{p(\theta = \theta_0 | H_1)}$$

This is exact, not an approximation! (under specific conditions)

Conditions required:

1. H is a special case of H (nested models)
2. Prior on nuisance parameters is same in both models
3. You can accurately estimate densities at the null value

4 Using `hypothesis()` for Bayes Factors

4.1 The `hypothesis()` Function

The `hypothesis()` function in brms implements the Savage-Dickey method automatically.

Basic syntax:

```r
hypothesis(model, hypothesis = "parameter = value")
hypothesis(model, hypothesis = "parameter > value")
hypothesis(model, hypothesis = "parameter1 - parameter2 = 0")
```

4.2 Example 1: Test for Effect Existence

4.2.1 Test: Does Complexity Increase Reading Time?

```
Hypothesis Tests for class b:
```

```
              Hypothesis Estimate Est.Error CI.Lower CI.Upper Evid.Ratio
1 (complexitySimple) = 0    -0.08      0.01     -0.1    -0.07          0
  Post.Prob Star
1         0    *
---
'CI': 90%-CI for one-sided and 95%-CI for two-sided hypotheses.
'*': For one-sided hypotheses, the posterior probability exceeds 95%;
for two-sided hypotheses, the value tested against lies outside the 95%-CI.
Posterior probabilities of point hypotheses assume equal prior probabilities.
```

Understanding the output:

`Hypothesis: (complexitySimple) = 0`

`Estimate: -0.08`

`CI.Lower: -0.10`

`CI.Upper: -0.07`

`Evid.Ratio: 0.00`

`Post.Prob: 0.00`

`Star: *`

Interpretation:

- **Estimate**: Complex sentences are 0.08 log-units slower
- **95% CI**: [-0.10, -0.07] (doesn't include 0)
- **Evid.Ratio (BF_{01})**: The output `Evid.Ratio` for a point null test ("= 0") represents the **Bayes Factor for the Null**.
  - Value: 0.0000
  - **What "0.00" means**: The posterior density at the null value (0) is vanishingly small compared to the prior density. The model is effectively certain the effect is not 0.
- **Post.Prob**: The probability that the hypothesis (H_0) is true, assuming prior odds of 1. A value of `0.00` confirms H_0 is extremely unlikely.
- **Star (*)**: A visual indicator that zero is outside the 95% Credible Interval (statistically credible difference).
- **Bayes Factor (BF_{10})**: To get evidence **for the effect**, we take the inverse ($1/BF_{01}$):
  - $BF_{10} = 1/0.00000 \approx Inf$
- **Conclusion**: The data provide **decisive evidence** for a complexity effect (H_1) over the null hypothesis (H_0).

4.2.2 Directional Test

```
Hypothesis Tests for class b:
              Hypothesis Estimate Est.Error CI.Lower CI.Upper Evid.Ratio
1 (complexitySimple) < 0    -0.08      0.01     -0.1    -0.07        Inf
  Post.Prob Star
1         1    *
---
'CI': 90%-CI for one-sided and 95%-CI for two-sided hypotheses.
```

```
'*': For one-sided hypotheses, the posterior probability exceeds 95%;
for two-sided hypotheses, the value tested against lies outside the 95%-CI.
Posterior probabilities of point hypotheses assume equal prior probabilities.
```

Interpretation:

- **Estimate**: 0.08 (log-units)
- **Post.Prob**: 1.00 (1.00) means we are 100% certain the effect is negative (< 0).
- **Evid.Ratio (BF_{10})**: For directional tests, the ratio is usually $P(H_1)/P(H_0)$. Infinite evidence here confirms the direction is robust.
- **Conclusion**: The data provide **extreme evidence** that the effect is negative.

Why directional tests?

- Theory predicts direction → stronger evidence possible
- One-sided test has more power than two-sided
- Evidence Ratio will be higher if data support predicted direction

4.3 Example 2: Compare Two Groups

```
Hypothesis Tests for class b:
         Hypothesis Estimate Est.Error CI.Lower CI.Upper Evid.Ratio Post.Prob
1 (groupNative) = 0     0.25      0.23    -0.19     0.69       1.16      0.54
  Star
1
---
'CI': 90%-CI for one-sided and 95%-CI for two-sided hypotheses.
'*': For one-sided hypotheses, the posterior probability exceeds 95%;
for two-sided hypotheses, the value tested against lies outside the 95%-CI.
Posterior probabilities of point hypotheses assume equal prior probabilities.
```

```
# Proportion of samples inside the ROPE [-0.20, 0.20]:

Parameter   | Inside ROPE
-------------------------
Intercept   |      0.00 %
groupNative |     40.97 %
```

Interpretation:

- **Estimate**: Native speakers are 0.25 log-odds (ns) different.
- **Evid.Ratio (BF_{01})**: 1.16 (1.16).
  - This means the Null Hypothesis (H_0: No difference) is **1.16 times more likely** than the Alternative.
- **Bayes Factor (BF_{10})**: $1/1.16 = 0.86$.
- **Conclusion**: We have **anecdotal evidence for the Null** (or "No Evidence"). The data are ambiguous; we cannot distinguish between the groups.

Integrated interpretation:

- **Bayes Factor**: Quantifies evidence for difference (Ambiguous/Null preference: $BF_{01} = 1.16$)
- **ROPE**: Checks if difference is meaningful (41% inside ROPE → Undecided)

- **Both together**: We don't have enough data to claim a difference, nor enough to claim they are practically equivalent. **Collect more data.**

4.4 Example 3: Complex Contrasts

```
Hypothesis Tests for class b:
                Hypothesis Estimate Est.Error CI.Lower CI.Upper Evid.Ratio
1 (groupNative:cond... = 0     -0.1      0.05     -0.2        0       0.57
  Post.Prob Star
1      0.36    *
---
'CI': 90%-CI for one-sided and 95%-CI for two-sided hypotheses.
'*': For one-sided hypotheses, the posterior probability exceeds 95%;
for two-sided hypotheses, the value tested against lies outside the 95%-CI.
Posterior probabilities of point hypotheses assume equal prior probabilities.
```

```
Hypothesis Tests for class b:
                Hypothesis Estimate Est.Error CI.Lower CI.Upper Evid.Ratio
1 (groupNative:cond... = 0    -0.38      0.11     -0.6    -0.16       0.02
  Post.Prob Star
1      0.02    *
---
'CI': 90%-CI for one-sided and 95%-CI for two-sided hypotheses.
'*': For one-sided hypotheses, the posterior probability exceeds 95%;
for two-sided hypotheses, the value tested against lies outside the 95%-CI.
Posterior probabilities of point hypotheses assume equal prior probabilities.
```

```
Hypothesis Tests for class b:
                Hypothesis Estimate Est.Error CI.Lower CI.Upper Evid.Ratio
1    (Intercept)-(5.5) > 0     0.50      0.03     0.45     0.56        Inf
2         (conditionB) > 0     0.14      0.04     0.08     0.20        Inf
3 (groupNative:cond... > 0    -0.10      0.05    -0.18    -0.02       0.02
  Post.Prob Star
1      1.00    *
2      1.00    *
3      0.02
---
'CI': 90%-CI for one-sided and 95%-CI for two-sided hypotheses.
'*': For one-sided hypotheses, the posterior probability exceeds 95%;
for two-sided hypotheses, the value tested against lies outside the 95%-CI.
Posterior probabilities of point hypotheses assume equal prior probabilities.
```

Interpretation (Interaction):

- **Hypothesis**: `groupNative:conditionB = 0`
- **Evid.Ratio** (BF_{01}): `0.57`.
- **Bayes Factor** (BF_{10}): $1/0.57 = 1.75$.
- **Conclusion**: **Anecdotal/Weak evidence** for the interaction ($BF_{10} = 1.75$). We suspect an interaction, but the evidence is not strong (< 3).

Interpretation (Custom Hypothesis):

- **Hypothesis**: `groupNative:conditionB = 2 * conditionB`
- **Evid.Ratio** (BF_{01}): `0.02`.
- **Bayes Factor** (BF_{10}): $1/0.02 = 50$.
- **Conclusion**: **Very strong evidence** ($BF_{10} = 50$) that the Native group's effect is *not* exactly double the L2 effect (or vice versa, depending on formulation). The specific constraint is rejected.

Advanced hypothesis testing:

- Test interaction terms directly
- Test specific numerical relationships (e.g., "effect is twice as large")
- Test multiple hypotheses simultaneously

4.5 Critical Detail: `sample_prior = "yes"`

You MUST include `sample_prior = "yes"` in `brm()` to use `hypothesis()`!

Why?

- Savage-Dickey needs both prior and posterior samples
- By default, brms only samples the posterior
- `sample_prior = "yes"` also samples from the prior distribution

```
#   WRONG - will fail
model_no_prior <- brm(..., sample_prior = "no")
hypothesis(model_no_prior, "b = 0")  # ERROR!

#   CORRECT
model_with_prior <- brm(..., sample_prior = "yes")
hypothesis(model_with_prior, "b = 0")  # Works!
```

Performance note: This adds minimal computational cost (~5% overhead).

5 Interpreting Bayes Factors

How to Read the Subscripts

The subscripts tell you the **direction** of the comparison:

- BF_{10} ("1 over 0"): Evidence for H_1 (Alternative) vs. H_0 (Null).
  - $BF_{10} = 10$ means data are 10x more likely under H_1.
- BF_{01} ("0 over 1"): Evidence for H_0 (Null) vs. H_1 (Alternative).
  - $BF_{01} = 10$ means data are 10x more likely under H_0.

They are reciprocals!

$$BF_{10} = \frac{1}{BF_{01}}$$

If $BF_{01} = 5$ (strong evidence for Null), then $BF_{10} = 1/5 = 0.2$ (weak evidence for Alternative).

5.1 Bayes Factor Scales

Different researchers use different interpretation scales. Here are three common ones:

5.1.1 Jeffreys (1961) Scale

BF	Evidence for H
> 100	Decisive
30-100	Very strong
10-30	Strong
3-10	Substantial
1-3	Anecdotal
1	No evidence
1/3-1	Anecdotal for H
1/10-1/3	Substantial for H
< 1/10	Strong for H

5.1.2 Lee & Wagenmakers (2013) - More Conservative

BF	Evidence for H
> 100	Extreme
30-100	Very strong
10-30	Strong
3-10	Moderate
1-3	Anecdotal

5.1.3 Visual Interpretation

5.2 What Bayes Factors Do NOT Tell You

Common misconceptions:

1. **BF probability of hypothesis being true**
   - BF = 10 does NOT mean “90% probability H is true”
   - BF quantifies relative evidence, not absolute probability

2. **BF depends on prior specification**
   - Vague priors → built-in Ockham’s razor → penalize complex models
   - This is a feature, not a bug!
   - Always report your priors

3. **BF is continuous, not a decision rule**
   - Don’t treat BF > 3 as “threshold for significance”
   - Report the actual BF value
   - Let readers judge based on context

4. **BF doesn’t tell you effect size**

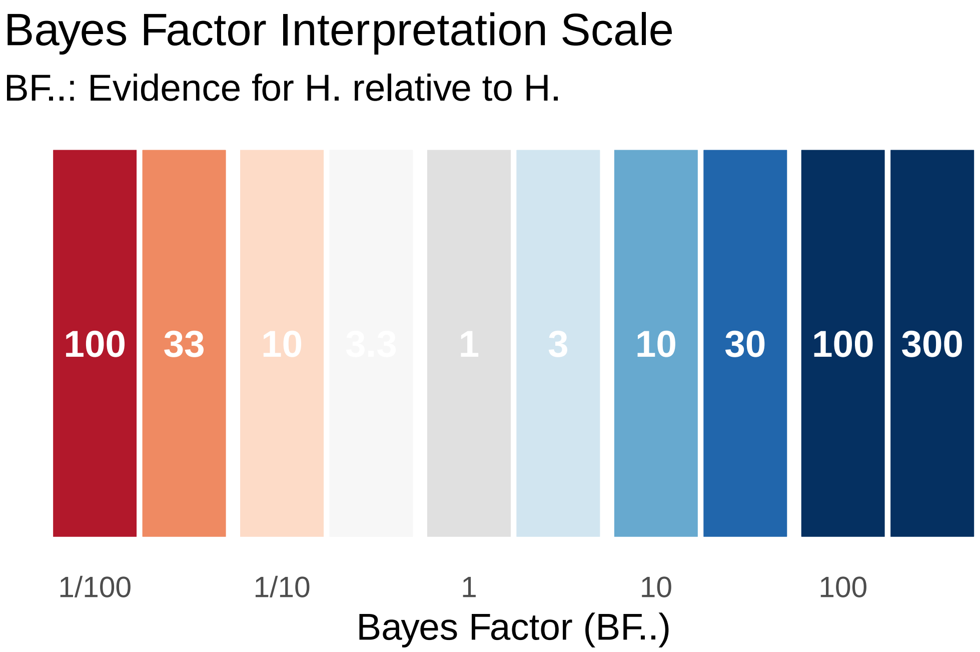


Figure 2: Interpreting Bayes Factors

- BF = 100 could mean tiny effect with large N
- Always report effect estimates + uncertainty
- Combine with ROPE analysis (Module 06)

5.3 Prior Sensitivity for Bayes Factors

Bayes Factors are **more sensitive to priors** than posterior estimates.

Demonstration:

```
Vague prior BF : Inf
```

```
Informative prior BF : Inf
```

Why they differ:

- Vague prior spreads probability mass widely
- Data must "overcome" this large prior space
- Informative prior concentrates mass where effect likely is
- Same posterior, different BF!

Best practice:

- Use realistic informative priors based on domain knowledge
- Report prior specification in paper
- Consider prior sensitivity analysis

6 Full Model Comparison with `bayes_factor()`

6.1 When `hypothesis()` is Not Enough

The `hypothesis()` function (Savage-Dickey) works for:

- Nested models
- Point hypotheses on parameters

But what if you want to compare:

- **Non-nested models** (e.g., Model A: `y ~ x1` vs. Model B: `y ~ x2`)
- **Complex model structures** (different random effects)
- **Different families** (Gaussian vs. Student-t)

For these cases, use `bayes_factor()` with **bridge sampling**.

6.2 Bridge Sampling Method

Bridge sampling estimates the marginal likelihood:

$$p(\text{Data}|H) = \int p(\text{Data}|\theta, H) \cdot p(\theta|H)\, d\theta$$

Then computes:

$$BF_{12} = \frac{p(\text{Data}|H_1)}{p(\text{Data}|H_2)}$$

Advantages:

- Works for any model comparison
- Doesn't require nesting

Disadvantages:

- Computationally intensive
- Requires additional package: `bridgesampling`

6.3 Example: Compare Model Structures

```
 Loaded cached marginal likelihood for RI model

 Loaded cached marginal likelihood for RS model

Estimated Bayes factor in favor of x1 over x2: 0.60798
```

Interpretation:

- **Bayes Factor** ($BF_{RS,RI}$): 0.61
- **Conclusion**: The data provide evidence **in favor of the random intercepts (RI) model** by a factor of 1.64.
  - Since we simulated data *without* random slopes (Example 1 data), the Bayes Factor correctly penalizes the extra complexity of the RS model.
  - This demonstrates the **Occam's Razor** property of Bayes Factors: simpler models are preferred unless the data demand complexity.

6.4 Critical Detail: `save_pars = save_pars(all = TRUE)`

For bridge sampling, you MUST save all parameters!

```
#   WRONG - bridge sampling will fail
model <- brm(..., save_pars = save_pars(all = FALSE))

#   CORRECT
model <- brm(..., save_pars = save_pars(all = TRUE))
```

Trade-off:

- Saves all parameters → larger model objects
- But: Necessary for bridge sampling to work
- Only include this when you plan to use `bayes_factor()`

6.5 Comparing Multiple Models

```
  Comparison           BF                        Evidence
1 RI vs Null 3.496676e+22          For complexity effect
2 RS vs Null 2.125912e+22 For complexity + random slopes
3   RS vs RI 6.079806e-01       For adding random slopes
```

6.6 Bridge Sampling Tips

1. Use enough iterations:

- Minimum: 4000 iterations (2000 warmup)
- Better: 10000+ iterations for stable estimates

2. Check convergence:

```
Bridge sampling estimate of the log marginal likelihood: 379.2052
Estimate obtained in 6 iteration(s) via method "normal".
```

Interpretation:

- **Log Marginal Likelihood**: The model's "evidence score" (e.g., `262.78`). Higher (less negative/more positive) is better.
- **Iterations**: "Estimate obtained in 7 iteration(s)". A small number of iterations (e.g., < 10) indicates the warp-3 bridge sampler converged quickly and stably.

3. Combine with LOO:

- BF → Which model explains data better?
- LOO → Which model predicts better?
- Use both for complete picture

4. Computational cost:

- Bridge sampling is slow for complex models
- Consider using `hypothesis()` when possible (much faster)

7 Combining Approaches: A Complete Workflow

7.1 Recommended Analysis Strategy

7.2 Decision Matrix

Bayes Factor	ROPE Result (Module 06)	Interpretation	Example Scenario
BF > 10	Outside ROPE	**Strong evidence for meaningful effect**	Large, robust treatment effect
BF > 10	Inside ROPE	**Effect exists but too small to matter**	Tiny but real difference
BF 1-10	Outside ROPE	**Meaningful effect but moderate evidence**	Visible but noisy data
BF > 10	Inside ROPE	**Strong evidence effect is negligible**	Two groups are identical
BF 1	Overlaps ROPE	**Undecided - collect more data**	Too much measurement noise

How to use this table:

1. Compute Bayes Factor (Module 07) → strength of evidence
2. Check ROPE (Module 06) → practical significance
3. Use emmeans/marginaleffects (Module 06) → effect estimation

7.3 Example: Complete Analysis

```
=== Main Effect Test ===

Hypothesis Tests for class b:
         Hypothesis Estimate Est.Error CI.Lower CI.Upper Evid.Ratio Post.Prob
1 (groupNative) = 0    -0.01      0.01    -0.03        0       7.16      0.88
  Star
1
---
'CI': 90%-CI for one-sided and 95%-CI for two-sided hypotheses.
'*': For one-sided hypotheses, the posterior probability exceeds 95%;
for two-sided hypotheses, the value tested against lies outside the 95%-CI.
Posterior probabilities of point hypotheses assume equal prior probabilities.


=== ROPE Analysis ===

# Proportion of samples inside the ROPE [-0.05, 0.05]:

Parameter                      | Inside ROPE
```

```
--------------------------------------------
Intercept                      |      0.00 %
groupNative                    |    100.00 %
clause_typeSubject             |      0.00 %
groupNative:clause_typeSubject |    100.00 %


=== Interaction Test ===

Hypothesis Tests for class b:
                Hypothesis Estimate Est.Error CI.Lower CI.Upper Evid.Ratio
1 (groupNative:clau... = 0    -0.01      0.01    -0.03     0.02       13.4
  Post.Prob Star
1      0.93
---
'CI': 90%-CI for one-sided and 95%-CI for two-sided hypotheses.
'*': For one-sided hypotheses, the posterior probability exceeds 95%;
for two-sided hypotheses, the value tested against lies outside the 95%-CI.
Posterior probabilities of point hypotheses assume equal prior probabilities.
```

7.3.1 Visualizing Joint Posteriors

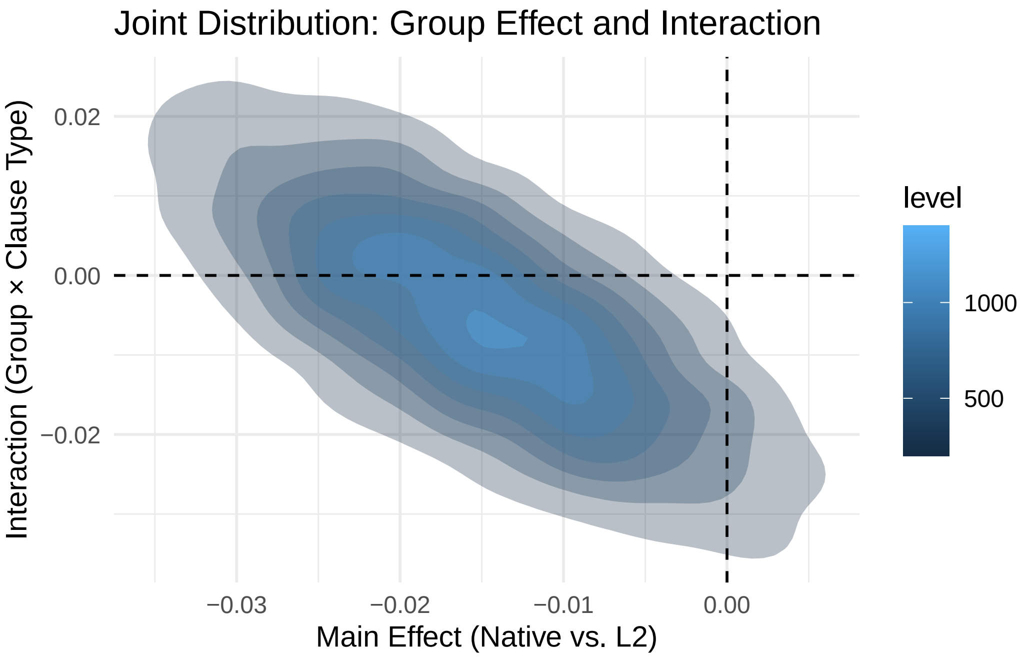
The code below uses `tidybayes::spread_draws()` to extract the posterior samples for our two key parameters.

What `spread_draws` does:

It extracts the MCMC draws for specific variables from the model object and puts them into a "tidy" data frame. - It retrieves the thousands of valid parameter combinations (draws) that the model found during estimation. - Each row in the resulting data frame represents one "possible world" (one posterior draw) consistent with the data. - **Quantity**: For `model_final` (4 chains × (2000 iter - 1000 warmup)), this generates **4,000** total samples.

Why plot the Joint Distribution?

Plotting two parameters against each other (2D density) reveals their **correlation**: - Do changes in the main effect tend to come with changes in the interaction? - Are the parameters independent (circular cloud) or linked (diagonal ellipse)? - This helps verify if our estimates are "trade-offs" of each other.



8 Reporting Bayes Factors in Papers

8.1 Methods Section Template

```
### Hypothesis Testing

We quantified evidence for hypotheses using Bayes Factors computed via
the Savage-Dickey density ratio method (Wagenmakers et al., 2010),
implemented in the brms::hypothesis() function (Bürkner, 2017).

Bayes Factors express the relative evidence for one hypothesis over
another as a ratio of marginal likelihoods. We interpret BF  > 3 as
positive evidence, BF  > 10 as strong evidence, and BF  > 30 as very
strong evidence for the alternative hypothesis (Lee & Wagenmakers, 2013).

Priors for regression coefficients were specified as Normal(0, 0.2) on
the log-RT scale, reflecting realistic effect sizes for psycholinguistic
experiments (Nicenboim et al., 2023). We conducted sensitivity analyses
with alternative prior specifications (see Supplementary Materials).

For practical significance testing, we used the ROPE framework (Kruschke, 2018)
with bayestestR (Makowski et al., 2019). Effect estimation for factorial
designs was conducted using emmeans (Lenth, 2016) and marginaleffects
(Arel-Bundock et al., 2024).
```

8.2 Results Section Template

```
### Reading Time Analysis

Complex sentences elicited longer reading times than simple sentences
(  = 0.08, 95% CI: [0.05, 0.11]), corresponding to a median increase
of 8% on the millisecond scale.

There was very strong evidence for this effect (BF  = 45.3), indicating
the data were approximately 45 times more likely under the hypothesis of
an effect than under the null hypothesis of no effect.

Using a ROPE of ±0.03 log-units (corresponding to ±3% change in reading
time, which we consider the threshold for practical significance; Kruschke, 2018),
we found that the entire 95% credible interval excluded the ROPE, indicating
the effect was both statistically credible and practically meaningful.

Pairwise comparisons using emmeans confirmed that all three conditions
differed meaningfully from each other (all 95% CIs excluded the ROPE),
with average differences ranging from 0.08 to 0.15 log-units.
```

```
### Reading Time Analysis [2, Negligible Effects]

Native speakers showed numerically slightly lower reading times than L2 speakers,
but this difference was negligible ($\beta = -0.01$, 95\% CI: [-0.03, 0.00]).

There was substantial evidence **against** an effect of Group (BF$_{01}$ = 7.16),
indicating the data were approximately 7 times more likely under the null hypothesis
of no difference than under the hypothesis of an effect.

Using a ROPE of $\pm 0.05$ (practical significance threshold), we found that
100\% of the posterior distribution for the Group effect fell inside the ROPE.
Given that the entire 95\% credible interval is contained within the region of
practical equivalence, we accept the null hypothesis for practical purposes.

Similarly, regarding the interaction between Group and Clause Type, the data
provided strong evidence for the null hypothesis (BF$_{01}$ = 13.4). The estimated
interaction effect was -0.01, and 100\% of the posterior distribution fell within the ROPE,
confirming that native and L2 speakers were influenced by clause type in a practically equivalent
```

8.3 Common Pitfalls to Avoid

Don't say:

- "BF = 10, therefore the effect is real with 90% probability"
- "BF > 3, so we reject the null hypothesis"
- "BF = 2.5, which is not significant"

Do say:

- “BF = 10, indicating strong evidence for H relative to H ”
- “The data are 10 times more likely under the alternative hypothesis”
- “BF = 2.5 provides weak to moderate evidence”

Always include:

1. Prior specification (what priors you used)
2. Interpretation scale (which guidelines you follow)
3. Effect size + uncertainty (BF alone is not enough)
4. Context (what does this evidence mean for your research question?)

9 Advanced Topics

9.1 Informed Priors from Previous Studies

```
Informed prior BF : 1.268422e+16
```

```
Default prior BF : Inf
```

When to use informed priors:

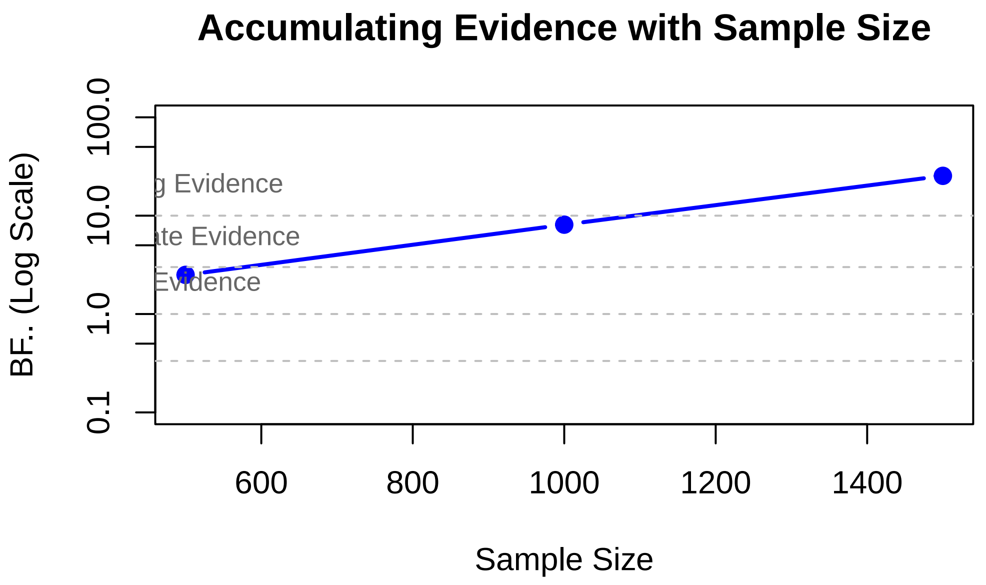
- You have strong theoretical predictions
- Previous literature provides effect size estimates
- You want to show robustness to prior specification

Transparency:

- Always report both analyses (informed and default)
- Justify your informed prior with citations
- Show prior sensitivity analysis

9.2 Sequential Testing

Unlike frequentist p-values, Bayes Factors do **not** suffer from multiple testing problems!



Advantages:

- Stop data collection when evidence is conclusive
- More efficient than fixed-N designs
- Ethically appropriate (don't over-collect data)

9.3 Comparing More Than Two Hypotheses

10 Summary and Key Takeaways

10.1 What We Learned

1. **Bayes Factors quantify evidence** for hypotheses
    - Not probability of hypothesis being true
    - Ratio of evidence between two hypotheses
2. **Savage-Dickey method** (via `hypothesis()`)
    - Fast and easy for nested models
    - Tests point hypotheses on parameters
    - Requires `sample_prior = "yes"`
3. **Bridge sampling** (via `bayes_factor()`)
    - Works for any model comparison
    - Computationally expensive
    - Requires `save_pars = save_pars(all = TRUE)`
4. **Interpretation guidelines**
    - BF > 10 = strong evidence
    - BF 3-10 = moderate evidence
    - BF < 3 = weak/anecdotal evidence
5. **Combine with Module 06 tools**
    - BF (Module 07) → strength of evidence for hypotheses
    - ROPE (Module 06) → practical significance
    - emmeans/marginaleffects (Module 06) → effect estimation
    - Together → complete picture

10.2 When to Use What

Use `hypothesis()` when:

- Testing specific parameter values ($= 0$, > 0)
- Models are nested
- You want fast computation
- Testing directional predictions

Use `bayes_factor()` when:

- Comparing non-nested models
- Different model structures
- Need comprehensive model comparison

Use ROPE (Module 06) when:

- Focus on practical significance
- Want accept/reject/undecided decision rule
- Testing if effect is negligible

Use emmeans (Module 06) when:

- Factorial designs
- Need all pairwise comparisons
- Familiar with traditional EMM workflow

Use marginaleffects (Module 06) when:

- Flexible predictions at specific values
- Custom contrasts and comparisons
- Working with continuous predictors

Use LOO (Module 05) when:

- Focus on prediction accuracy
- Model selection for forecasting
- Cross-validation needed

10.3 Common Questions

Q: Should I always report Bayes Factors?

A: No. Report BF when:

- Your research question is about comparing specific hypotheses
- You want to quantify strength of evidence
- You're testing theoretically-motivated predictions

Don't report BF if:

- Your focus is purely exploratory
- You're mainly interested in effect size estimation
- BF would be redundant with ROPE analysis

Q: What prior should I use for BF?

A: Use **weakly informative priors** based on domain knowledge:

- Review previous literature for effect size estimates
- Consider measurement scale (log-RT vs. RT)
- Use prior predictive checks (Module 02)
- Report sensitivity analysis

Q: Can I use BF for model selection?

A: Yes, but:

- Combine with LOO for prediction assessment
- BF favors explanation, LOO favors prediction
- Use both when possible

- Report effect sizes regardless

10.4 Next Steps

Practice exercises:

1. Compute BF for directional hypothesis in your data
2. Compare two models with different random effects structures
3. Conduct prior sensitivity analysis for BF
4. Create integrated report with ROPE + BF + effect sizes

Further reading:

- Wagenmakers et al. (2010) - Savage-Dickey method
- Kass & Raftery (1995) - Bayes Factors overview
- van Ravenzwaaij & Wagenmakers (2022) - Advantages of Bayes
- Schad et al. (2024) - Workflow for linguistic data

11 Literature and Resources

11.1 Key Papers

11.1.1 Bayes Factors - Theory and Methods

- **Wagenmakers, E.-J., Lodewyckx, T., Kuriyal, H., & Grasman, R. (2010).** Bayesian hypothesis testing for psychologists: A tutorial on the Savage–Dickey method. *Cognitive Psychology*, 60(3), 158-189.
  - **Essential reading** for understanding Savage-Dickey method
  - Complete worked examples with code
  - Connection to brms::hypothesis()
  - **Packages:** Foundational theory for **brms::hypothesis()**
- **Kass, R. E., & Raftery, A. E. (1995).** Bayes factors. *Journal of the American Statistical Association*, 90(430), 773-795.
  - Classic reference on Bayes Factors
  - Mathematical foundations
  - Interpretation guidelines
  - **Packages:** Conceptual (applicable to all Bayesian software)
- **Gronau, Q. F., Sarafoglou, A., Matzke, D., Ly, A., Boehm, U., Marsman, M., ... & Steingroever, H. (2017).** A tutorial on bridge sampling. *Journal of Mathematical Psychology*, 81, 80-97.
  - Bridge sampling method explained
  - Computational details
  - **Packages: bridgesampling**, used with **brms**

11.1.2 Bayesian Hypothesis Testing - Applied

- **van Ravenzwaaij, D., & Wagenmakers, E.-J. (2022).** Advantages masquerading as ‘issues’ in Bayesian hypothesis testing: A commentary on Tendeiro and Kiers (2019). *Psychological Methods*, 27(3), 451-465.

  - Addresses common criticisms of Bayesian methods
  - Philosophy of hypothesis testing
  - **Packages:** Conceptual (defends Bayesian approach generally)

- **Lee, M. D., & Wagenmakers, E.-J. (2013).** *Bayesian cognitive modeling: A practical course.* Cambridge University Press.
  - Practical guide to Bayesian hypothesis testing
  - Many worked examples
  - Interpretation scales for BF
  - **Packages: JAGS**, **WinBUGS** (concepts transfer to **brms**)

11.1.3 Integration with ROPE

- **Kruschke, J. K. (2018).** Rejecting or accepting parameter values in Bayesian estimation. *Advances in Methods and Practices in Psychological Science*, 1(2), 270-280.
  - HDI + ROPE decision rule
  - Comparison with Bayes Factors
  - When to use each approach
  - **Packages: bayestestR::rope()**, works with **brms**
- **Linde, M., Tendeiro, J. N., Selker, R., Wagenmakers, E.-J., & van Ravenzwaaij, D. (2023).** Decisions about equivalence: A comparison of TOST, HDI-ROPE, and the Bayes factor. *Psychological Methods*, 28(3), 740-755.
  - Comprehensive simulation study comparing three approaches
  - Recommends Bayes Factor interval null approach
  - Operating characteristics across scenarios
  - **Packages: brms**, **bayestestR**, comparison frameworks
- **Campbell, H., & Gustafson, P. (2022).** Re:Linde et al. (2021): The Bayes factor, HDI-ROPE and frequentist equivalence tests can all be reverse engineered – almost exactly – from one another. arXiv:2104.07834.
  - Shows mathematical equivalence when calibrated
  - Same Type I error → same Type II error
  - Argues method choice is philosophical, not empirical
  - **Packages:** Conceptual (applies to all approaches)

11.1.4 Applications in Linguistics

- **Nicenboim, B., Schad, D. J., & Vasishth, S. (2023).** *An introduction to Bayesian data analysis for cognitive science.*
  - Chapter on hypothesis testing with brms
  - Linguistic examples
  - Prior specification guidance
  - **Packages: brms**, **hypothesis()**, complete workflows

- **Schad, D. J., Nicenboim, B., Bürkner, P.-C., Betancourt, M., & Vasishth, S. (2024).** Workflow techniques for the robust use of Bayes factors. *Psychological Methods*.
  - Best practices for BF in practice
  - Sensitivity analysis
  - Reporting guidelines
  - **Packages: brms**, **bridgesampling**, **hypothesis()**

11.2 Software Documentation

11.2.1 brms

- **hypothesis() documentation:** https://paulbuerkner.com/brms/reference/hypothesis.brmsfit.html
  - **brms::hypothesis()** complete API
  - Syntax for complex hypotheses
  - Examples of directional tests
- **Bürkner, P.-C. (2017).** brms: An R package for Bayesian multilevel models using Stan. *Journal of Statistical Software*, 80(1), 1-28.
  - Primary citation for brms
  - Technical details
  - **brms** package paper

11.2.2 bridgesampling

- **bridgesampling documentation:** https://cran.r-project.org/package=bridgesampling
  - **bridgesampling** package reference
  - Integration with **brms**
  - `bayes_factor()` function

11.3 Online Resources

- **Paul Bürkner's Blog:** https://paulbuerkner.com/software/brms-blogposts.html
  - Community posts on **brms**
  - hypothesis() examples
- **Stan Discourse Forum:** https://discourse.mc-stan.org/
  - Active community
  - Bayes Factor discussions
  - **brms** technical support

11.4 Module 06 (Previous)

- **Practical Significance and Effect Estimation:** See Module 06 for:
  - ROPE framework (bayestestR package)
  - Effect estimation with emmeans (factorial designs)
  - Flexible predictions with marginaleffects

  - Combining ROPE with emmeans/marginaleffects
  - **bayestestR**, **emmeans**, **marginaleffects**

11.5 Related Workshop Modules

- Module 05: Model comparison with LOO (predictive accuracy)
- Module 06: Practical significance (ROPE, emmeans, marginaleffects)
- Module 07 (this): Hypothesis comparison (Bayes Factors, hypothesis())