Posterior Predictive Checks: Grammaticality Judgment Example

Bayesian Mixed Effects Models with brms for Linguists

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1 Posterior Predictive Checks for Binary Data

After fitting your model, validate that it generates data similar to what you observed.

1.1 Why Posterior Predictive Checks Matter

Posterior predictive checks answer: "If I were to generate new data from my fitted model, would it look like my actual data?" This validates that your model has captured the essential structure of your data.

For binary outcomes (correct/incorrect, yes/no), we focus on: - **Proportion of successes** (mean of 0/1 outcomes) - **Accuracy by condition** - **Calibration** (predicted probabilities match observed frequencies)

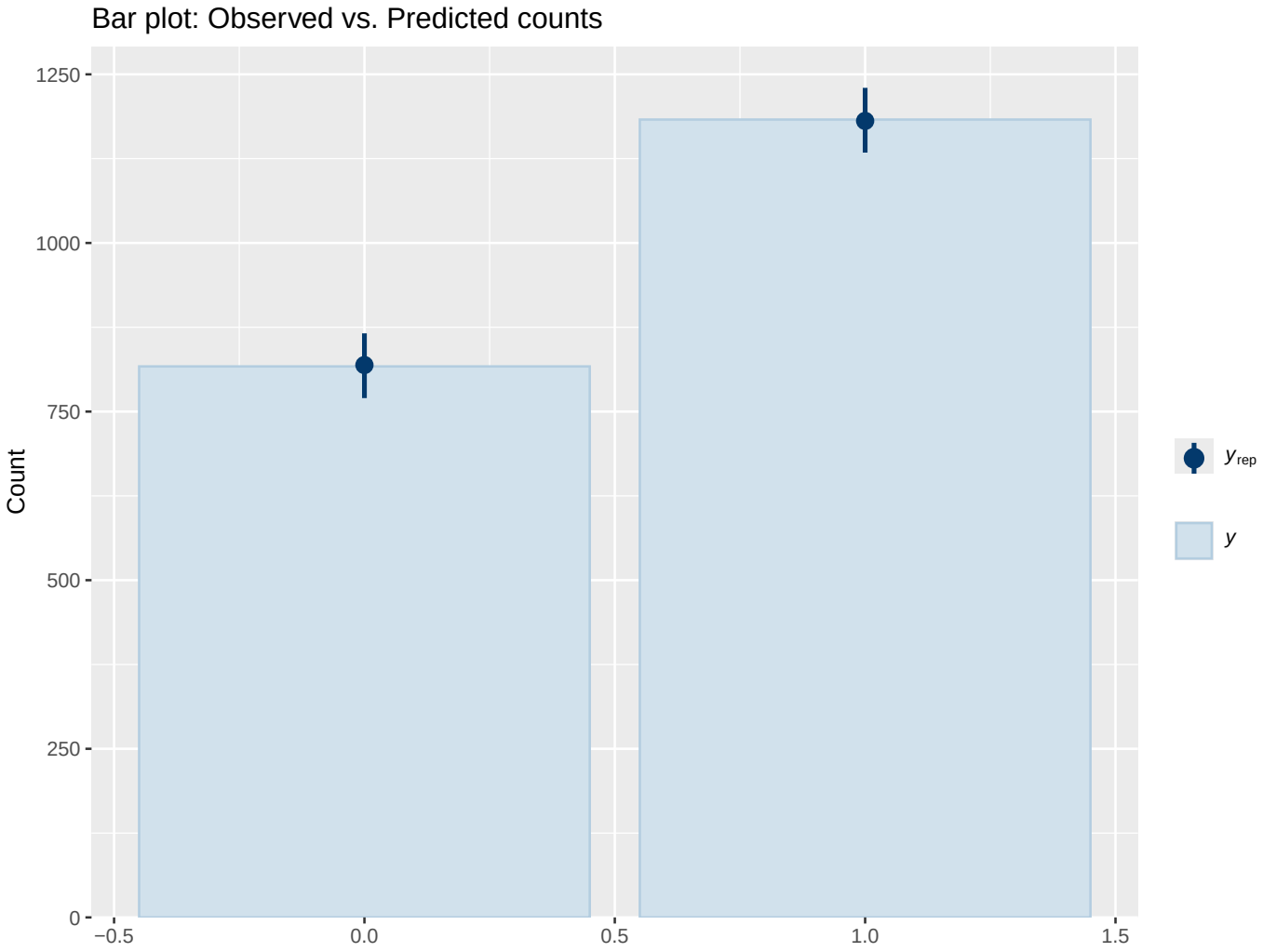
1.2 Setup

1.3 Load or Fit Model

```
Loading saved model from: fits/fit_gram.rds


=== Model Summary ===

 Family: bernoulli
  Links: mu = logit
Formula: correct ~ condition + (1 + condition | subject) + (1 | item)
   Data: gram_data (Number of observations: 2000)
  Draws: 2 chains, each with iter = 1000; warmup = 500; thin = 1;
         total post-warmup draws = 1000

Multilevel Hyperparameters:
~item (Number of levels: 2)
              Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
sd(Intercept)     0.21      0.24     0.01     0.91 1.03      102      119

~subject (Number of levels: 25)
                           Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS
sd(Intercept)                  0.10      0.07     0.00     0.26 1.00      213
sd(conditionB)                 0.13      0.09     0.01     0.34 1.00      254
cor(Intercept,conditionB)     -0.06      0.44    -0.81     0.76 1.01      427
                           Tail_ESS
sd(Intercept)                   268
sd(conditionB)                  560
cor(Intercept,conditionB)       635

Regression Coefficients:
           Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
Intercept      0.17      0.18    -0.34     0.46 1.02      125       64
conditionB     0.34      0.09     0.17     0.51 1.01      970      657

Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
and Tail_ESS are effective sample size measures, and Rhat is the potential
scale reduction factor on split chains (at convergence, Rhat = 1).
```

1.4 Posterior Predictive Checks for Binary Data

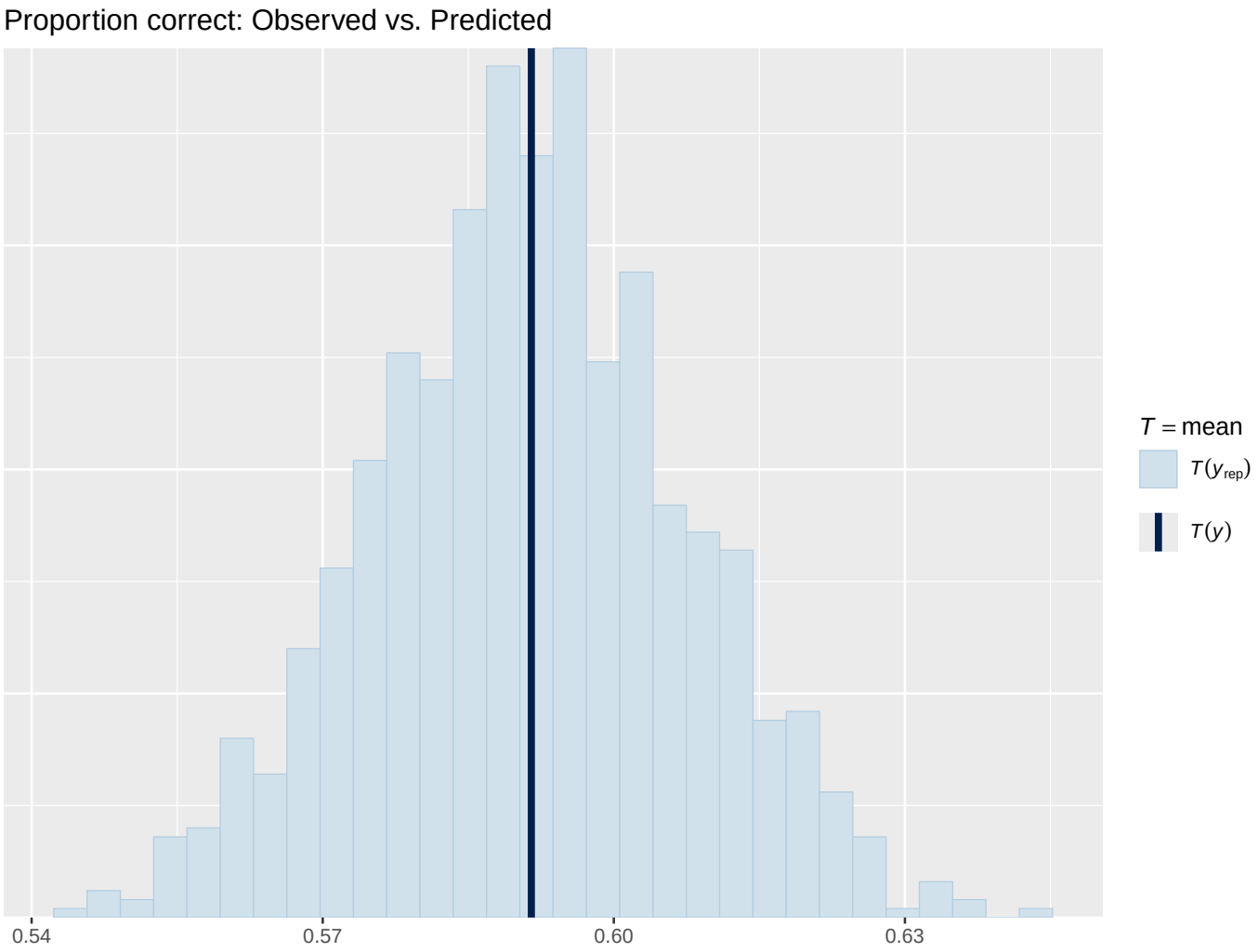
1.4.1 Bar Plot Comparison



Interpretation:

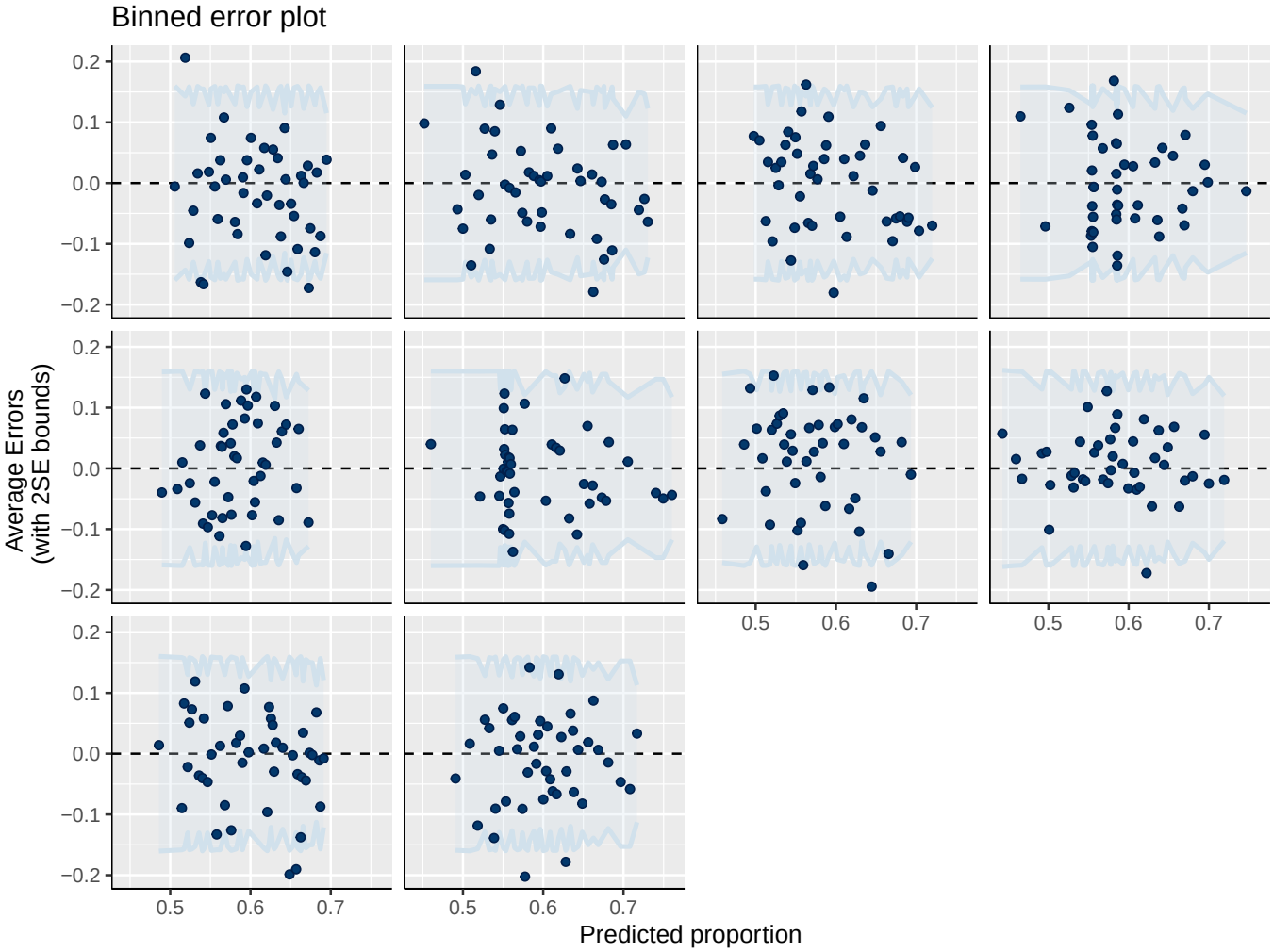
- Bars show frequency of 0s (incorrect) and 1s (correct)
- Light blue bars = observed data
- Dark lines = posterior predictions
- Should see good overlap between observed and predicted frequencies

1.4.2 Check Proportion Correct



The mean of binary data = proportion of 1's (proportion correct). This check verifies that the model's predicted accuracy matches the observed accuracy.

1.4.3 Error Plot for Discrete Data



Interpretation:

- Shows prediction error patterns
- Points should scatter around zero
- Systematic patterns suggest model misspecification

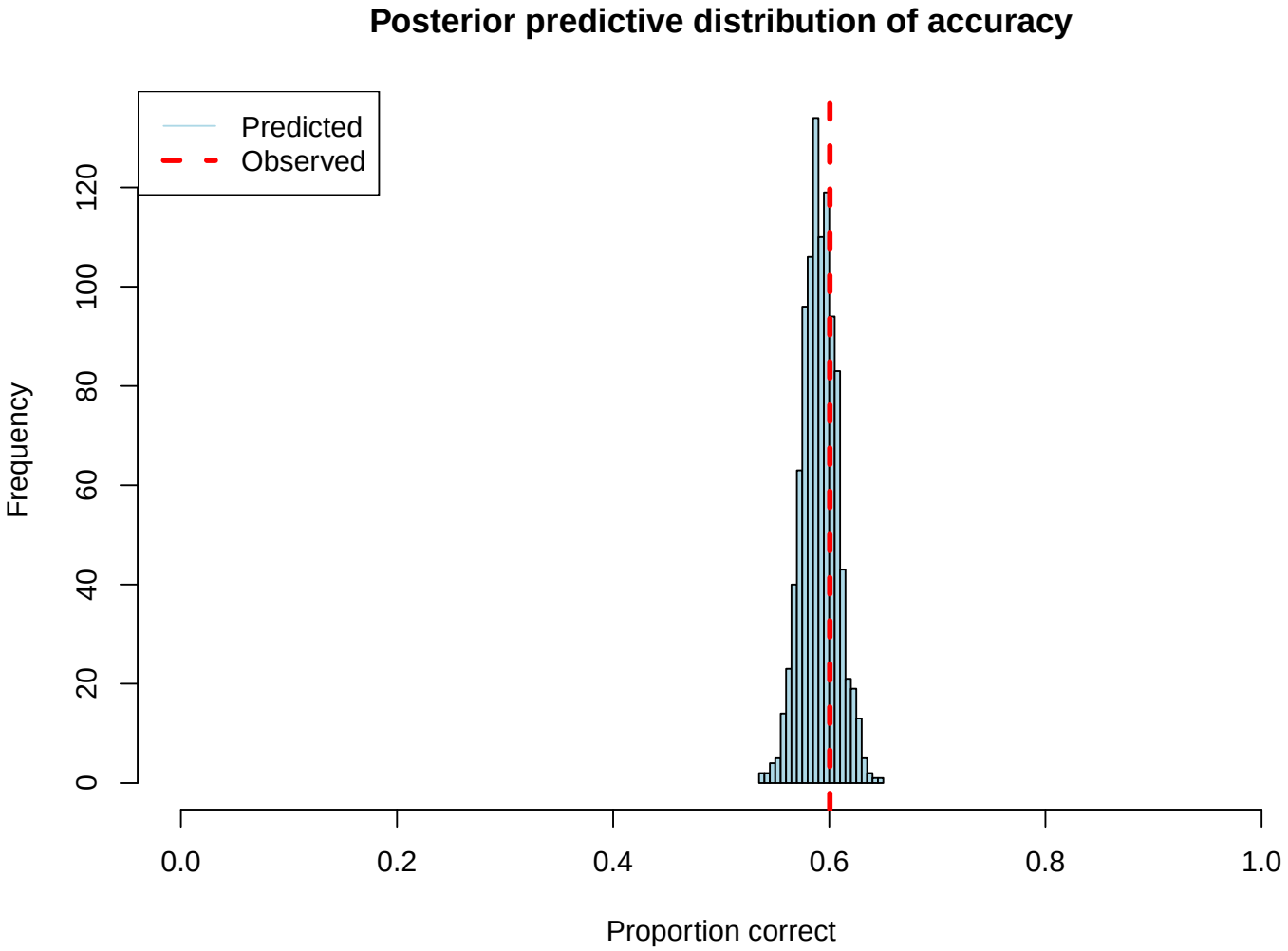
1.5 Extract and Analyze Posterior Predictions

1.5.1 Predicted Binary Outcomes

```
[1] 1000 2000


Dimensions of posterior predictions:

Draws: 1000

Observations: 2000


Overall Accuracy:
```

```
Observed: 0.601

Predicted (median): 0.591

95% CI: 0.56 0.624
```

1.5.2 Distribution of Predicted Accuracy



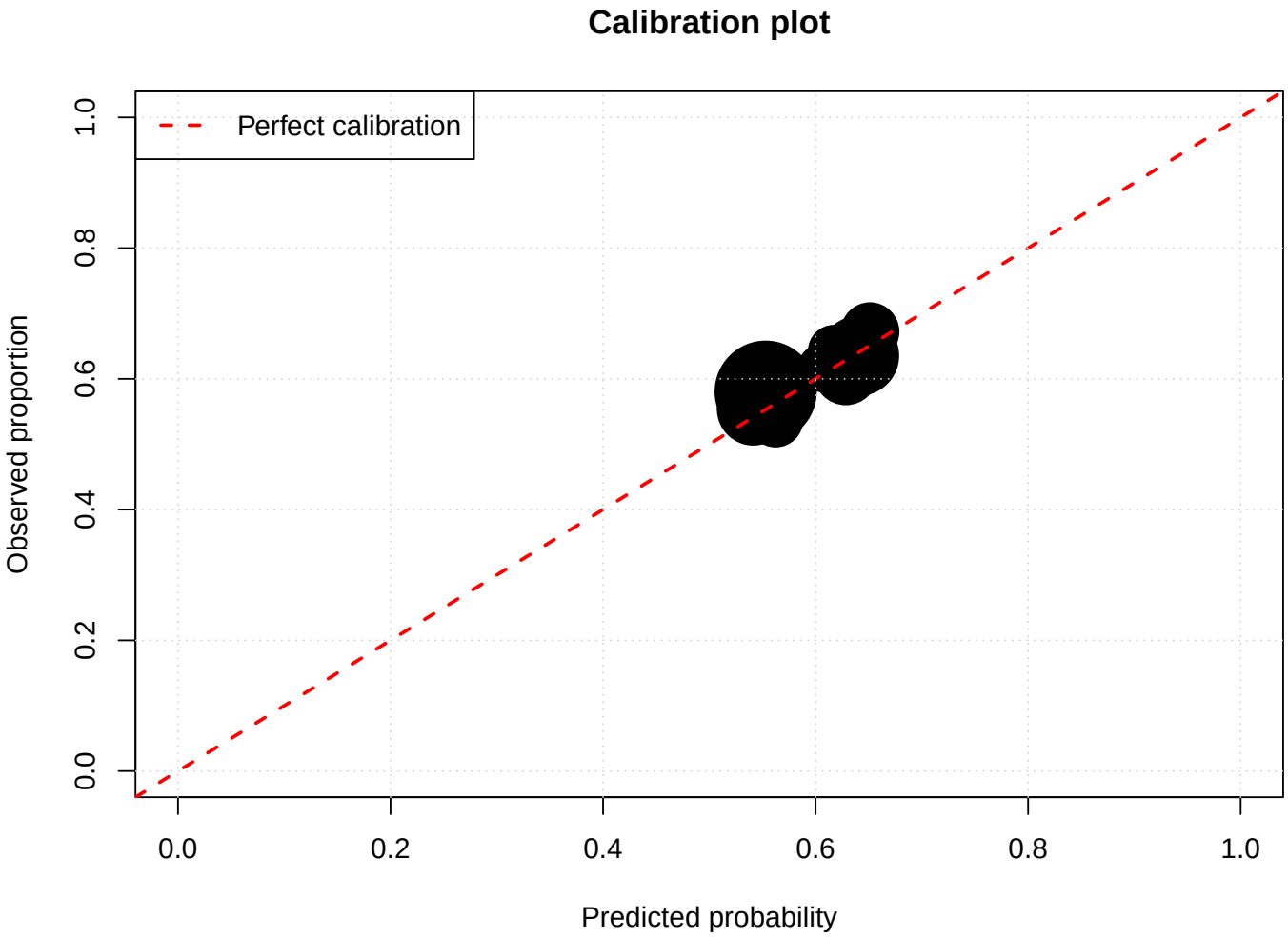

```
 Observed accuracy within 95% posterior predictive interval
```

1.6 Expected Value (Probability) Summaries

```
[1] 1000 2000

Posterior expected probabilities:

Dimensions: 1000 2000

Example: First observation
```

```
Observed outcome: 0

Predicted P(correct): 0.501 0.554 0.614
```

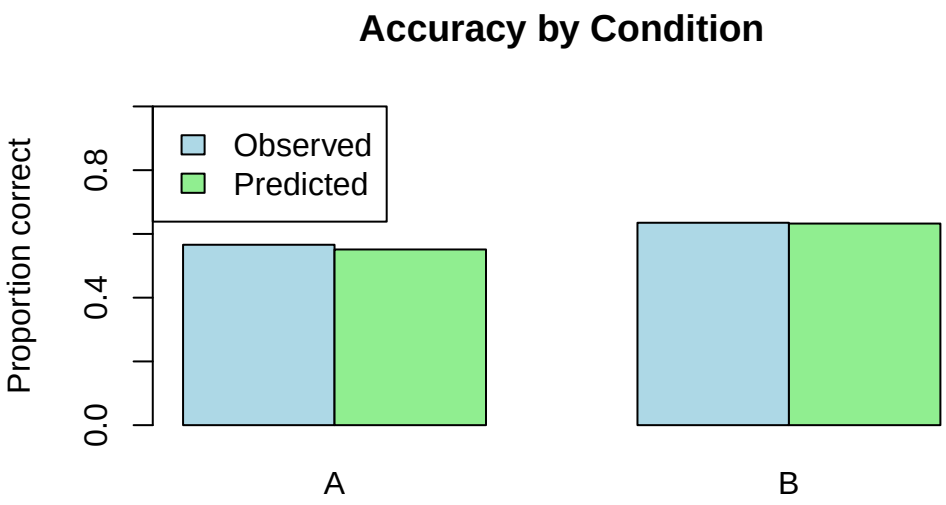
1.6.1 Calibration Check




```
Calibration:

Points should fall near the diagonal line.

Point size proportional to number of observations.
```

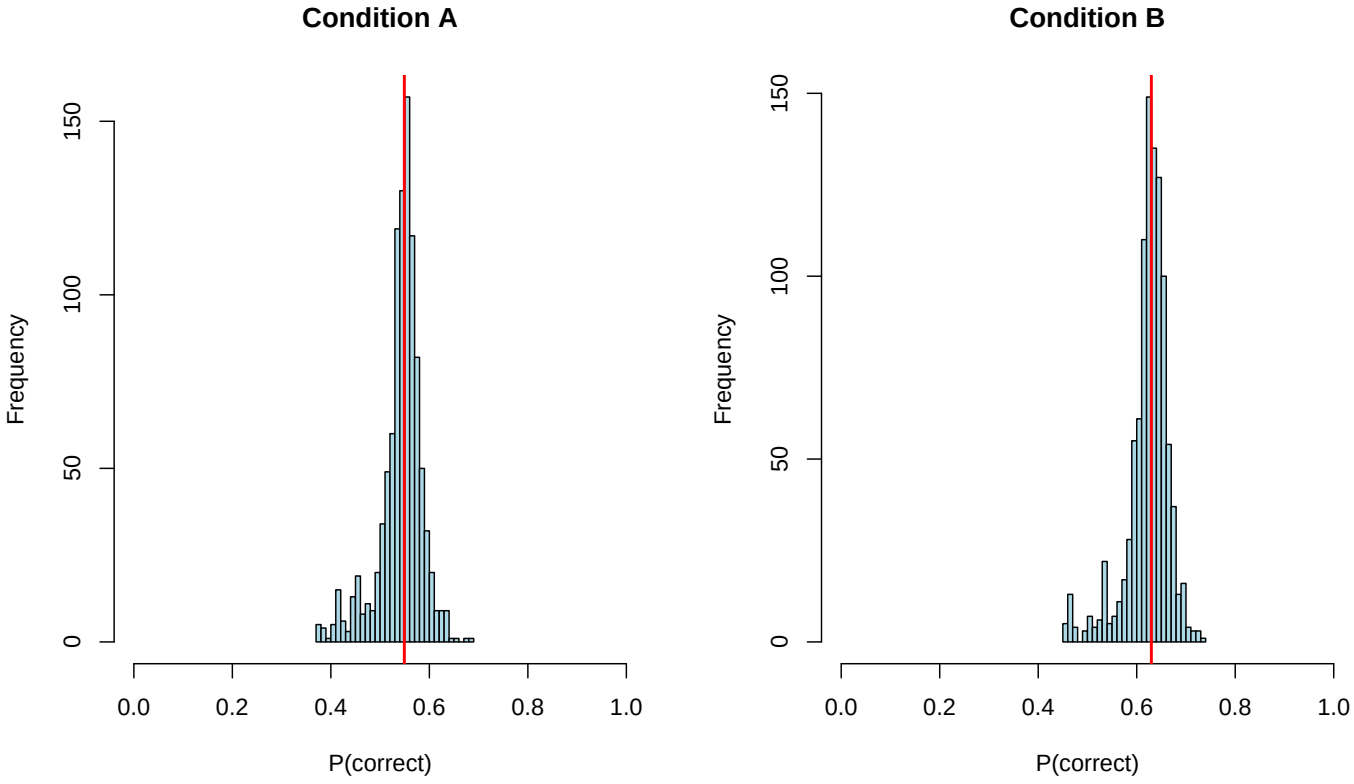
1.7 Check by Condition

```
Accuracy by Condition:

# A tibble: 2 x 4
  condition accuracy     n pred_accuracy
  <chr>        <dbl> <int>         <dbl>
1 A            0.566  1000         0.551
2 B            0.635  1000         0.633
```



1.7.1 Posterior Predictions by Condition

```
Population-level P(correct) by condition:

           A     B
2.5%   0.417 0.501
50%    0.549 0.630
97.5%  0.614 0.690
```



1.8 Summary

1.8.1 Key Diagnostics Checked

☒ **Bar plot** - Observed counts match posterior predictions
☒ **Proportion correct** - Overall accuracy captured correctly
☒ **Calibration** - Predicted probabilities align with observed frequencies
☒ **By condition** - Model captures differences between conditions
☒ **Expected values** - Posterior probabilities are reasonable

1.8.2 Interpreting Binary Model Checks

- **Observed proportion correct** should be near the central tendency of the posterior predictions
- If observed is far from predicted → model isn't capturing the accuracy pattern
- Common issues:
  - Forgetting interactions
  - Wrong random effect structure
  - Not accounting for item-level variation

1.8.3 Common Problems and Solutions

Problem	Diagnosis	Solution
Poor calibration	Points far from diagonal	Add predictors, check formula
Misses condition effects	Different accuracy by condition not captured	Add condition × random effect interaction
Predictions too certain	Predicted probs near 0 or 1	Check priors, may be too strong
Predictions too uncertain	Predicted probs all near 0.5	Add more structure, informative priors

1.8.4 Next Steps

If posterior predictive checks reveal problems:

1. **Adjust model formula** - Add missing predictors or interactions
2. **Revise priors** - May be too restrictive or too vague
3. **Check random effects structure** - Subjects and items may vary in unexpected ways
4. **Consider response time cutoffs** - Fast guesses vs. thoughtful responses

```
R version 4.4.1 (2024-06-14)
Platform: x86_64-pc-linux-gnu
Running under: Ubuntu 22.04.5 LTS

Matrix products: default
BLAS:   /usr/lib/x86_64-linux-gnu/openblas-pthread/libblas.so.3
LAPACK: /usr/lib/x86_64-linux-gnu/openblas-pthread/libopenblasp-r0.3.20.so;  LAPACK version 3.10.0

locale:
 [1] LC_CTYPE=en_US.UTF-8       LC_NUMERIC=C
 [3] LC_TIME=en_US.UTF-8        LC_COLLATE=en_US.UTF-8
```

```
 [5] LC_MONETARY=en_US.UTF-8    LC_MESSAGES=en_US.UTF-8
 [7] LC_PAPER=en_US.UTF-8       LC_NAME=C
 [9] LC_ADDRESS=C               LC_TELEPHONE=C
[11] LC_MEASUREMENT=en_US.UTF-8 LC_IDENTIFICATION=C

time zone: Etc/UTC
tzcode source: system (glibc)

attached base packages:
[1] stats     graphics  grDevices utils     datasets  methods   base

other attached packages:
 [1] bayesplot_1.14.0 lubridate_1.9.3  forcats_1.0.0    stringr_1.5.1
 [5] dplyr_1.1.4      purrr_1.0.2      readr_2.1.5      tidyr_1.3.1
 [9] tibble_3.2.1     ggplot2_4.0.0    tidyverse_2.0.0  brms_2.23.0
[13] Rcpp_1.0.13

loaded via a namespace (and not attached):
 [1] gtable_0.3.6          tensorA_0.36.2.1      xfun_0.54
 [4] QuickJSR_1.8.1        inline_0.3.21         lattice_0.22-6
 [7] tzdb_0.4.0            vctrs_0.6.5           tools_4.4.1
[10] generics_0.1.3        stats4_4.4.1          parallel_4.4.1
[13] fansi_1.0.6           pkgconfig_2.0.3       Matrix_1.7-0
[16] checkmate_2.3.3       RColorBrewer_1.1-3    S7_0.2.0
[19] distributional_0.5.0  RcppParallel_5.1.11-1 lifecycle_1.0.4
[22] compiler_4.4.1        farver_2.1.2          Brobdingnag_1.2-9
[25] tinytex_0.53          codetools_0.2-20      htmltools_0.5.8.1
[28] yaml_2.3.10           pillar_1.9.0          StanHeaders_2.32.10
[31] bridgesampling_1.1-2  abind_1.4-8           nlme_3.1-164
[34] posterior_1.6.1.9000  rstan_2.32.7          tidyselect_1.2.1
[37] digest_0.6.37         mvtnorm_1.3-3         stringi_1.8.4
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[55] matrixStats_1.5.0     emmeans_2.0.0         gridExtra_2.3
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[67] plyr_1.8.9            R6_2.5.1
```